Progress in Electromagnetic Gyrokinetic Simulations of Microturbulence in Toroidal Geometry with Kinetic Electrons

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ABSTRACT

The effects of fully kinetic electrons, including both trapped and passing electrons, on turbulence and transport in tokamaks are studied using a 3-dimensional gyrokinetic particle code that uses field-line-following coordinates in a flux-tube and the split-weight scheme for the electrons (I. Manuiskiy and W. W. Lee, Phys. Plasmas 7(5), 1381 (2000)). The problem of high plasma beta in kinetic electron simulations (Y. Chen and S. E. Parker, Phys. Plasmas 8(5), 2001) is revisited. The problem arises from the large term, $\beta m_i/m_e A_\parallel$ in the Ampere's law (in dimensionless form), $[-\nabla_\perp^2 + \beta\frac{m_i}{m_e}]A_\parallel = \beta\delta j_\parallel$. When $\beta m_i/m_e$ well exceeds $k_\perp^2$, finite β effects cannot be accurately simulated using previous algorithms. By evaluating the $\beta m_i/m_e$ term using the same discrete representation of marker distribution and the same scattering operation as that used for δf, finite β effects for the three branches of wave – Alfvén wave, drift wave and ITG – are all accurately observed for $\beta m_i/m_e \geq 20$ in a 3-D shearless geometry. Finite β stabilization of toroidal ITG and the kinetic ballooning mode are also observed. The electron-ion collisions are found to be stabilizing for both instabilities.

Split-weight Scheme

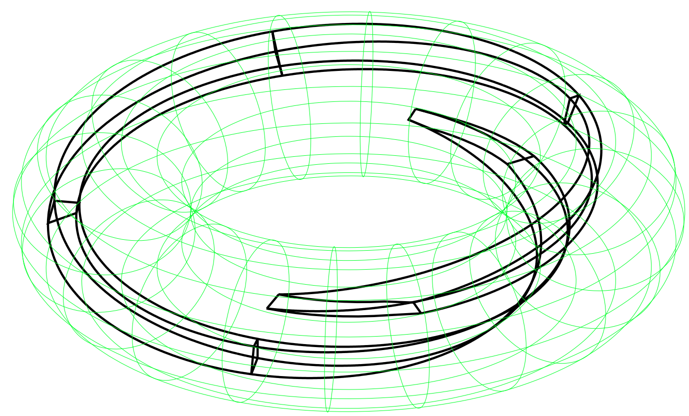
- Perturbation $\mathbf{E}_\perp = \nabla_\perp \phi$, $E_\parallel = \nabla_\parallel \phi - \dfrac{\partial A_\parallel}{\partial t}$, $\delta \mathbf{B}_\perp = \nabla A_\parallel \times \mathbf{b}$
- Use $p_\parallel = v_\parallel + \dfrac{q}{m} A_\parallel$ as a coordinate to eliminate $\dfrac{\partial A_\parallel}{\partial t}$. (Hahm'88)
- $f_e = f_{\mathrm{Me}} + \varepsilon_g e\phi \dfrac{f_M}{T} + h$.
- Quasi-neutrality: $\tilde{\phi} = \Sigma_{\mathbf{k}} \Gamma_0(b)\phi_k \exp(i\mathbf{k}\cdot\mathbf{x})$

$$n_{0\mathrm{i}} \frac{q_i^2}{T_i}(\phi - \tilde{\phi}) + \varepsilon_g n_{0\mathrm{e}} \frac{e^2}{T_e}\phi = \delta\bar{n}_i - \delta n_{\mathrm{eh}}$$

- Equation for $\partial\phi/\partial t$

$$n_{0\mathrm{i}} \frac{q_i^2}{T_i}\left(\frac{\partial \phi}{\partial t} - \frac{\partial \tilde{\phi}}{\partial t}\right) = -\nabla \cdot \int f_i \mathbf{v}_{\mathrm{Gi}} d\mathbf{v} + \nabla \cdot \int f_e \mathbf{v}_{\mathrm{Ge}}\, d\mathbf{v}$$

- Ampere's law $\left(-\nabla_\perp^2 + \frac{\omega_{\mathrm{pe}}^2}{c^2}\right) A_\parallel = \mu_0 \left(u_{\parallel\mathrm{i}} - u_{\parallel\mathrm{e}}\right)$



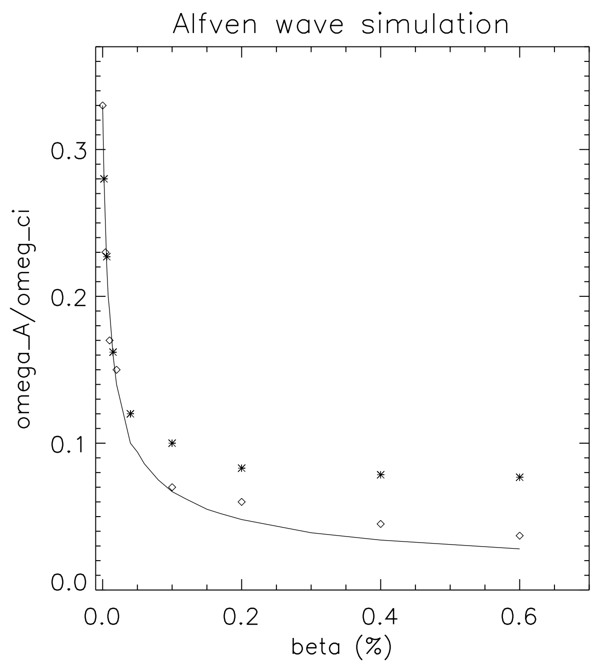
- $x = r - r_0, \;\; y = \frac{r_0}{q_0}(q\theta - \zeta), \quad z = q_0 R_0 \theta.$
- $l_z = 2\pi q_0 R_0$. Periodic in x-y, toroidal bc's.
- Predictor-Corrector (no electron sub-cycling and orbit averaging)
- Fourier method for gyrokinetic quasi-neutrality condition and Ampere's law
- 1D domain decomposition in z with domain cloning at each z. (C. Kim and S. E. Parker, 2000)

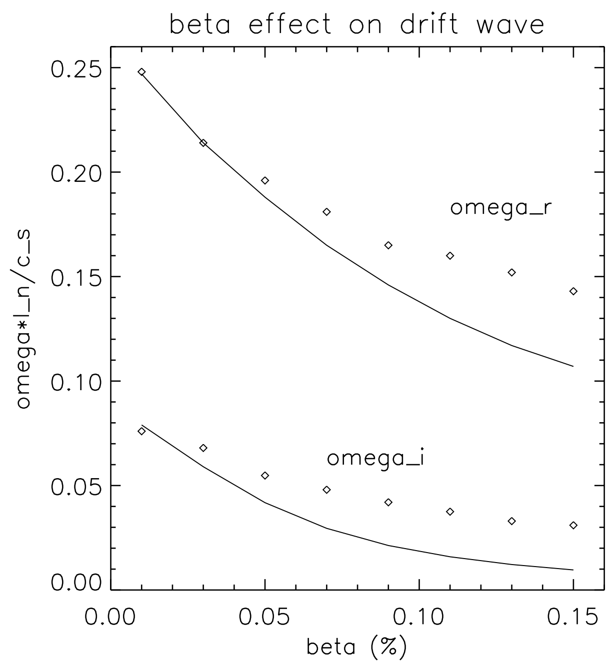
High β Problem and Solution

- For $k_\perp \rho_i \leq 1$ the $\frac{\omega_{\rm pe}^2}{c^2} = \beta_e \frac{m_i}{m_e}$ term much larger than the $k_\perp^2$ term in the Ampere's law.
- This term should be exactly cancelled linearly by a corresponding part of $u_{\|\rm e}$. However, this cancellation is difficult to attain, since $u_{\|\rm e}$ comes from particles and contains the effects of finite particle number and finite particle size

$$u_{\|\rm e}(\mathbf{x}) = \sum_j w_{\rm ej}\, v_{\|\rm j}\, S(\mathbf{x}_j - \mathbf{x})$$

w_e electron weight. S is the particle shape. $\mathbf{x}$ location of the grid point, $\mathbf{x}_j$ the particle position.





- The problem is not caused by the split-weight scheme. Using $v_\parallel$ formulation with the same split-weight scheme does not solve the problem. However, $v_\parallel$ formulation with a different split-weight scheme (W. W. Lee, 2001) was demonstrated in low-dimensionality simulations to be free from this difficulty. It is not clear whether this is true in 3-D simulations.
- $v_\parallel$ formulation with finite differencing $\frac{\partial A_\parallel}{\partial t}$, even time centered, leads to numerical instability.

Solution

- Evaluate $\frac{\omega_{\rm pe}^2}{c^2} A_\parallel$ using the same discrete marker representation and scattering operation as that used for $u_{\parallel \rm e}$, both in the Ampere's law and in the equation for $\frac{\partial \phi}{\partial t}$.

$$\left(\frac{\omega_{\rm pe}^2}{c^2} A_\parallel\right)(\mathbf{x}) = \int v_\parallel \, f_M(v) \, d^3\mathbf{v} = \frac{\omega_{\rm pe}^2}{c^2} \sum_j A_\parallel(\mathbf{x}_j) \, S(\mathbf{x}_j - \mathbf{x})$$

- $A_\parallel$ unknown prior to solving Ampere's law. Use iteration

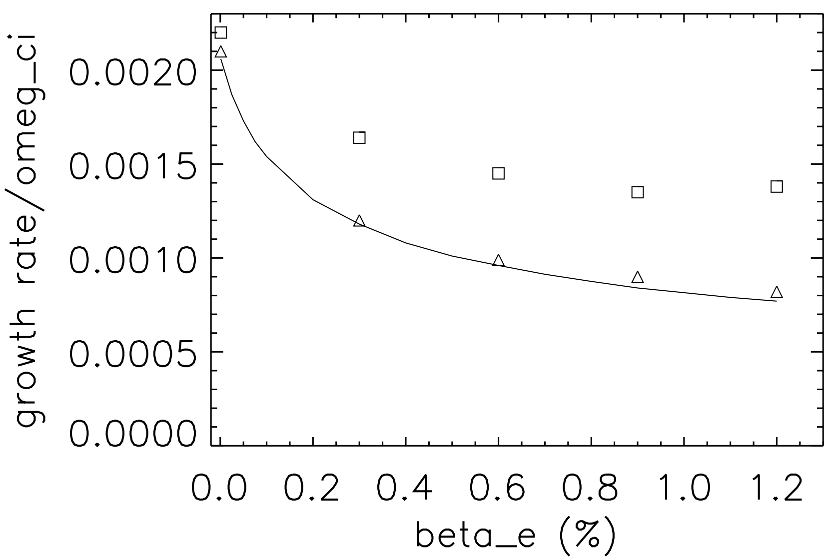
$$\left(-\nabla_\perp^2 + \frac{\omega_{\rm pe}^2}{c^2}\right) A_\parallel^{n+1} = \beta_e \left(u_{\parallel \rm i} - u_{\parallel \rm e}\right) + \frac{\omega_{\rm pe}^2}{c^2}\left(A_\parallel^n - \sum_j A_\parallel^n(\mathbf{x}_j) S(\mathbf{x}_j - \mathbf{x})\right)$$

  number of iteration $5 \sim 7$. This also works for the nonlinear term $\delta n_e A_\parallel$ in the Ampere's law.
- It is found that similar technique for the $\varepsilon_g n_{0\rm e} \frac{e^2}{T_e} \phi$ term in the quasi-neutrality condition is not necessary, since $\varepsilon_g \leq 1$.

Finite β Effects on Slab ITG - Good Agreement Between Simulation and Dispersion Relation

Slab dispersion relation $-k_\perp^2 \frac{k_\parallel}{\omega}(M_i - M_e - {k'}_\perp^2) = \beta(N_e - N_i)({k'}_\perp^2 - L_i + L_e)$ with

$L_i = (\Omega - \Omega_{\mathrm{Ti}})\Gamma_0 + \Omega_{\mathrm{Ti}}\Gamma_*$, $M_i = -\Gamma_0(1+\zeta_i Z_i) + (\frac{3}{2}\Omega_{\mathrm{Ti}}\Gamma_0 - \Omega_{\mathrm{Ti}}\Gamma_* - \Omega\Gamma_0)\zeta_i Z_i - \Omega_{\mathrm{Ti}}\Gamma_0\zeta_i^2(1+\zeta_i Z_i)$, $N_i = -\frac{\omega}{k_\parallel}[\Omega\Gamma_0(1+\zeta_i Z_i) + (-\frac{3}{2}\Omega_{\mathrm{Ti}}\Gamma_0 + \Gamma_0 + \Omega_{\mathrm{Ti}}\Gamma_*)(1+\zeta_i Z_i) + \Omega_{\mathrm{Ti}}\Gamma_0(\frac{1}{2} + \zeta_i^2 + \zeta_i^3 Z_i)]$, $L_e = \Omega$, $M_e = 1 - (\Omega - \Omega_{\mathrm{Te}}/2 - 1)\zeta_e Z_e(\zeta_e) - \Omega_{\mathrm{Te}}\zeta_e^2(1+\zeta_e Z_e)$, $N_e = -\frac{\omega}{k_\parallel}[\Omega_{\mathrm{Te}}(\frac{1}{2} + \zeta_e^2(1+\zeta_e Z_e)) + (\Omega - \Omega_{\mathrm{Te}}/2 - 1)(1+\zeta_e Z_e)]$, $\kappa_n = -\frac{dn_0}{dx}/n_0$, $\kappa_{\mathrm{T}\alpha} = -\frac{dT_{0\alpha}}{dx}/T_0$, $\Omega = \kappa_n k_y/\omega$, $\Omega_{\mathrm{T}\alpha} = \kappa_{\mathrm{T}\alpha} k_y/\omega$, $\Gamma_0 = \Gamma_0(b) = \Gamma_0(k_\perp^2 v_{\mathrm{ti}}^2/\Omega_i^2)$, $\Gamma_* = \Gamma_0 - b(\Gamma_0 - \Gamma_1)$, ${k'}_\perp^2 = 1 - \Gamma_o(b)$. $\zeta_\alpha = \omega/\sqrt{2}k_\parallel v_{\mathrm{t}\alpha}$. $Z_\alpha = Z(\zeta_\alpha)$ the plasma dispersion function.



$k_x\rho_i = 0.2$, $k_y\rho_i = 0.4$, $k_\parallel\rho_i = 7.14 \times 10^{-4}$, $\kappa_T\rho_i = 0.2$, $\kappa_n\rho_i = 0.04$, $\kappa_{\mathrm{Te}}\rho_i = 0$, $32 \times 32 \times 32$, 262 144 e's and i's, $\varepsilon_g = 0.5$,

$l_x \times l_y \times l_z = 32\rho_i \times 32\rho_i \times 8796\rho_i$, $\triangle t\, \omega_{\mathrm{ci}} = 2$

Electron-ion Collisions

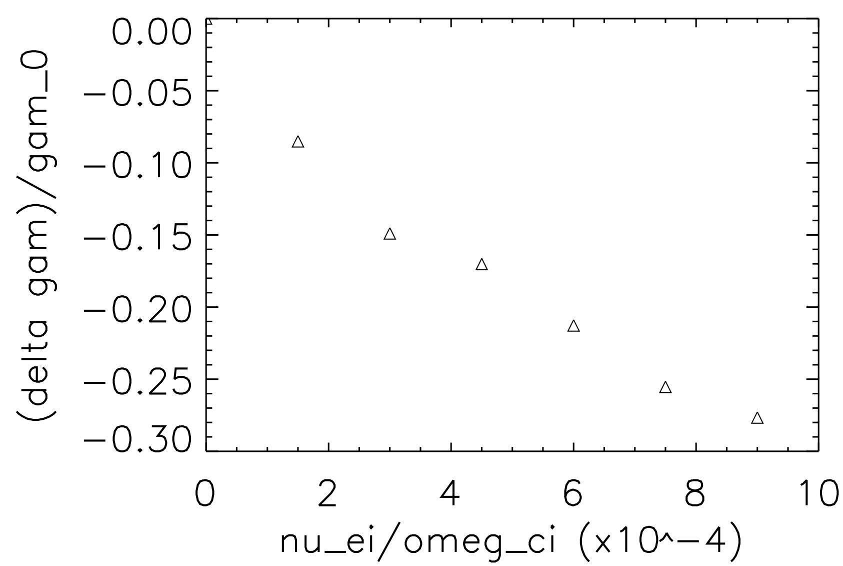
- Lorentzian operator

$$C_L(f) = -\nu_e(v)\frac{\partial}{\partial\lambda}(1-\lambda^2)\frac{\partial}{\partial\lambda}f, \quad \nu_e(v) = 2\pi e^2(ne^2)_i \ln\lambda / m_e^2 v^3$$

- $C_L(\varepsilon_g e\phi f_M/T) = 0$. $C_L(h)$ implemented with Monte-Carlo method at both predictor and corrector steps.

$$\lambda_{\text{new}} = \lambda_{\text{old}}(1 - 2\nu_e\triangle t) \pm \left[\left(1-\lambda_{\text{old}}^2\right)2\nu_e\triangle t\right]^{\frac{1}{2}}$$

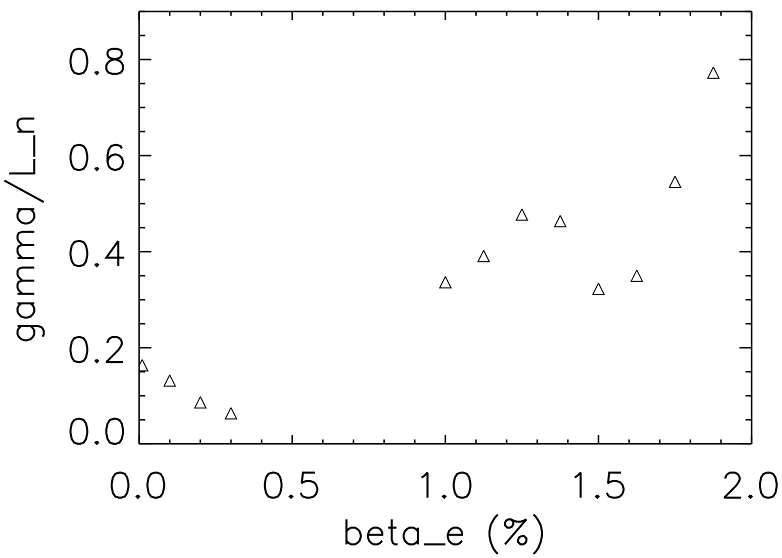
where $\pm$ means equal probability of $+$ or $-$ (Boozer and Kuo-Petravic, '81). $\triangle t = dt$ for corrector step and $\triangle t = 2dt$ for predictor step, dt is the time step.



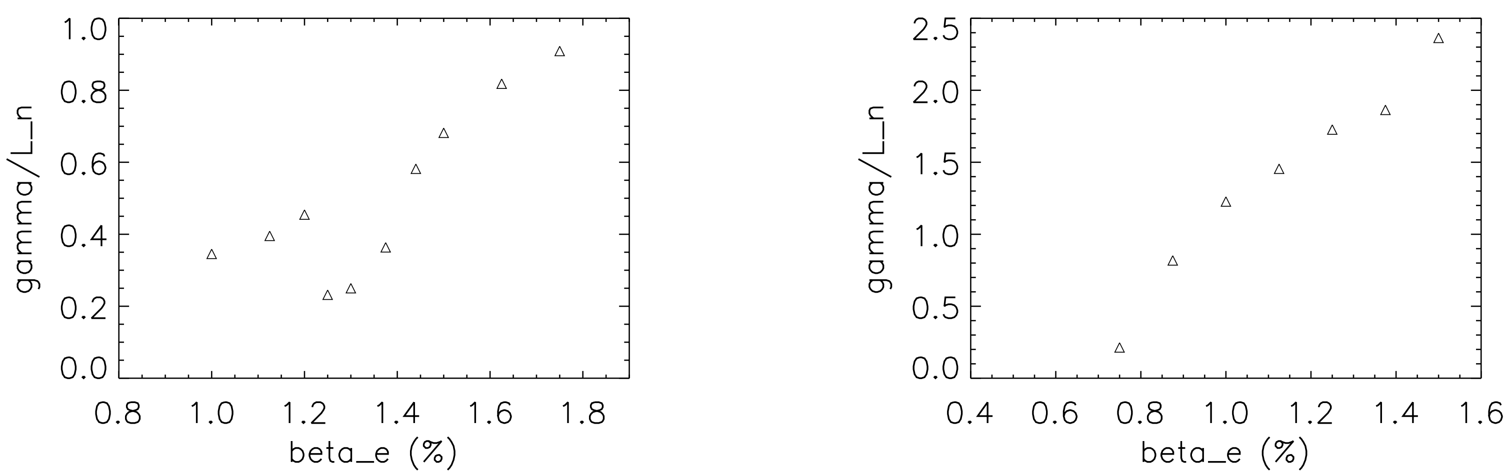
DIII-D Base case $k_y\rho_i = 0.3$ ITG mode growth rate vs. collision rate.

Finite β Stabilization of Toroidal ITG and the Kinetic Ballooning Mode

- Both finite β effects are observed with the new algorithm. Old code numerically unstable for large β.
- Hybrid model with zero mass electrons shows finite β effects on ITG and kinetic ballooning modes (S. Parker, Y. Chen and C. Kim, 2000; G. Zhao and L. Chen, 2002). However, 3-D hybrid simulation has difficulty with $k_{\|}\rho_i = 0$ component, does not approach the adiabatic limit even though trapped electron effects are neglected.
- e-i collisions have strong effects on the KBMs. With $\nu_{\mathrm{ei}}/\omega_{\mathrm{ci}} = 3 \times 10^{-4}$, there is a sudden drop in the growth rate of KBM near $\beta_e \approx 1.5\%$.
- Across this critical β_e the eigenmode $\phi(x, y, z)$ changes its structure along the field line (z direction). For $\beta_e < \beta_c$ ϕ even in z with maximum at $z = 0$ (similar to ITG), for $\beta_e > \beta_c$ ϕ odd in z. $A_{\|}$ is always opposite to ϕ.
- While eigenfunction ϕ for ITG is dominated by $k_x = 0$ component, KBM shows more complicated radial structure.
- Without collisions no abrupt change in growth rate seen.

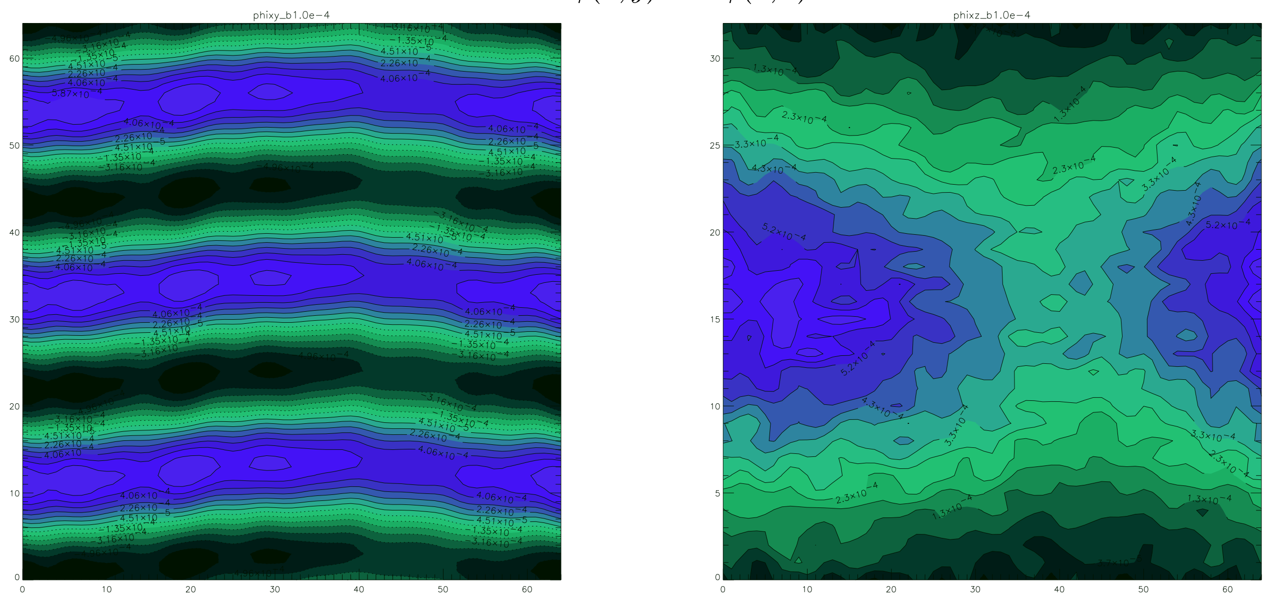


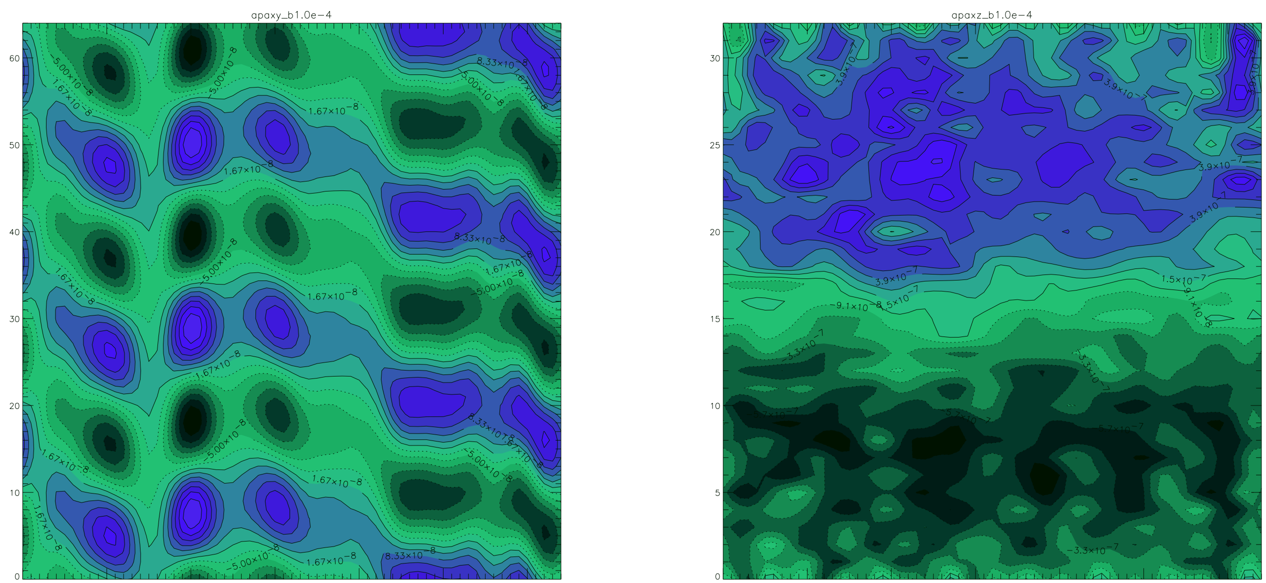
γ vs. β_e for $k_y\rho_i = 0.3$ mode



γ vs. β_e for $k_y\rho_i = 0.2$ mode, with and without collisions

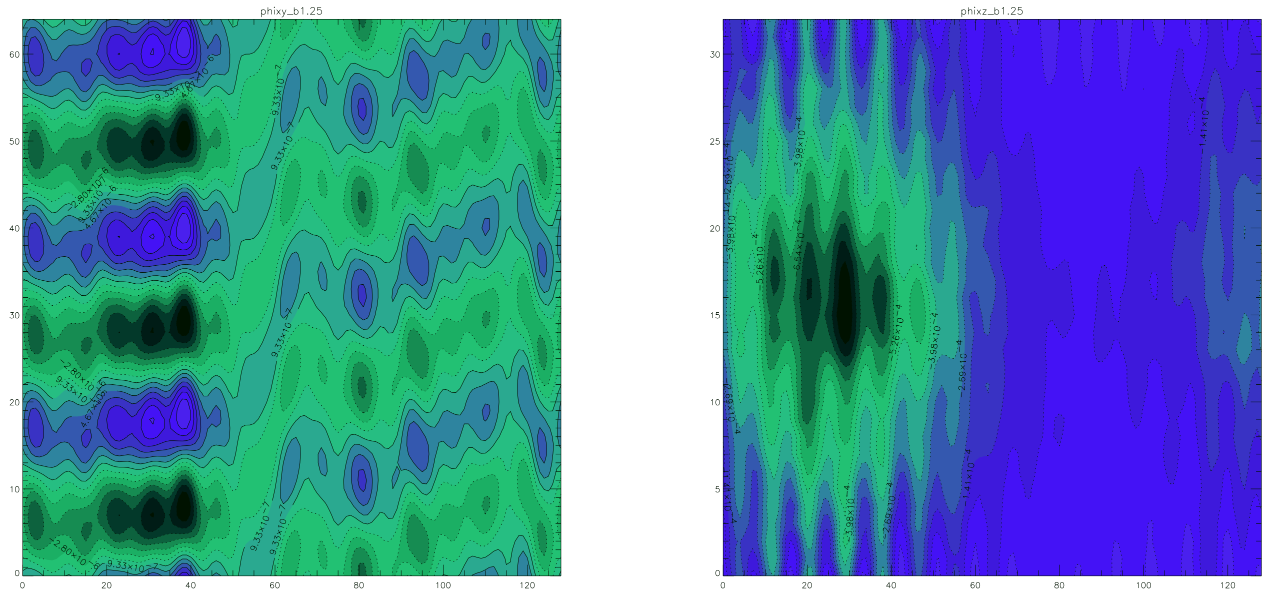
ITG $\phi(x,y)$ and $\phi(x,z)$

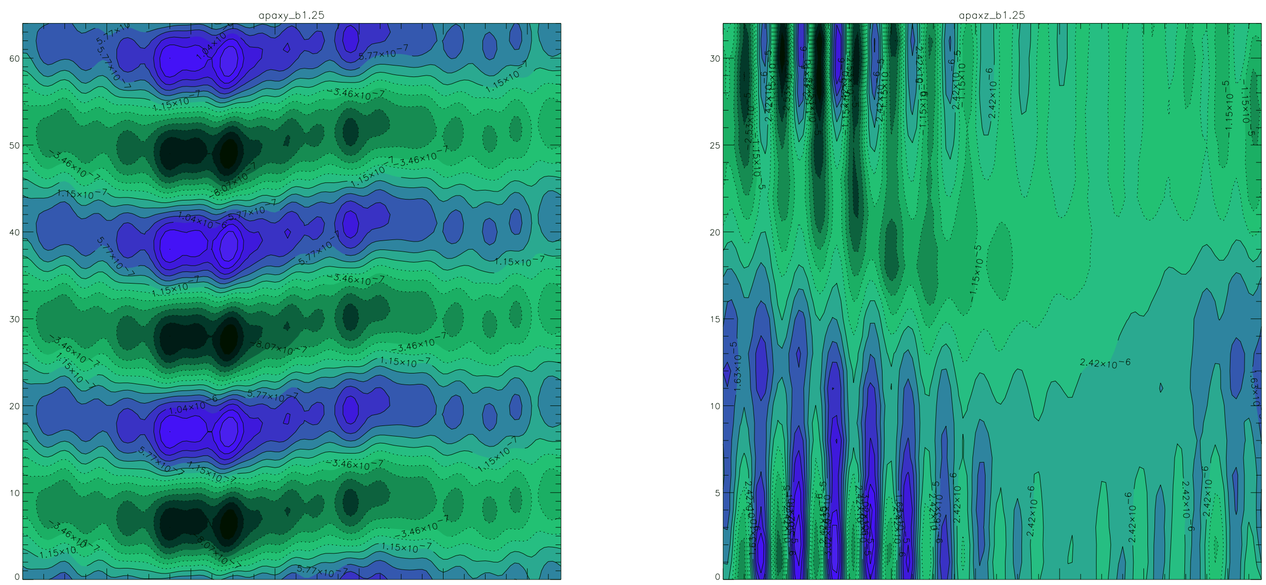




ITG $A_{\|}(x,y)$ and $A_{\|}(x,z)$

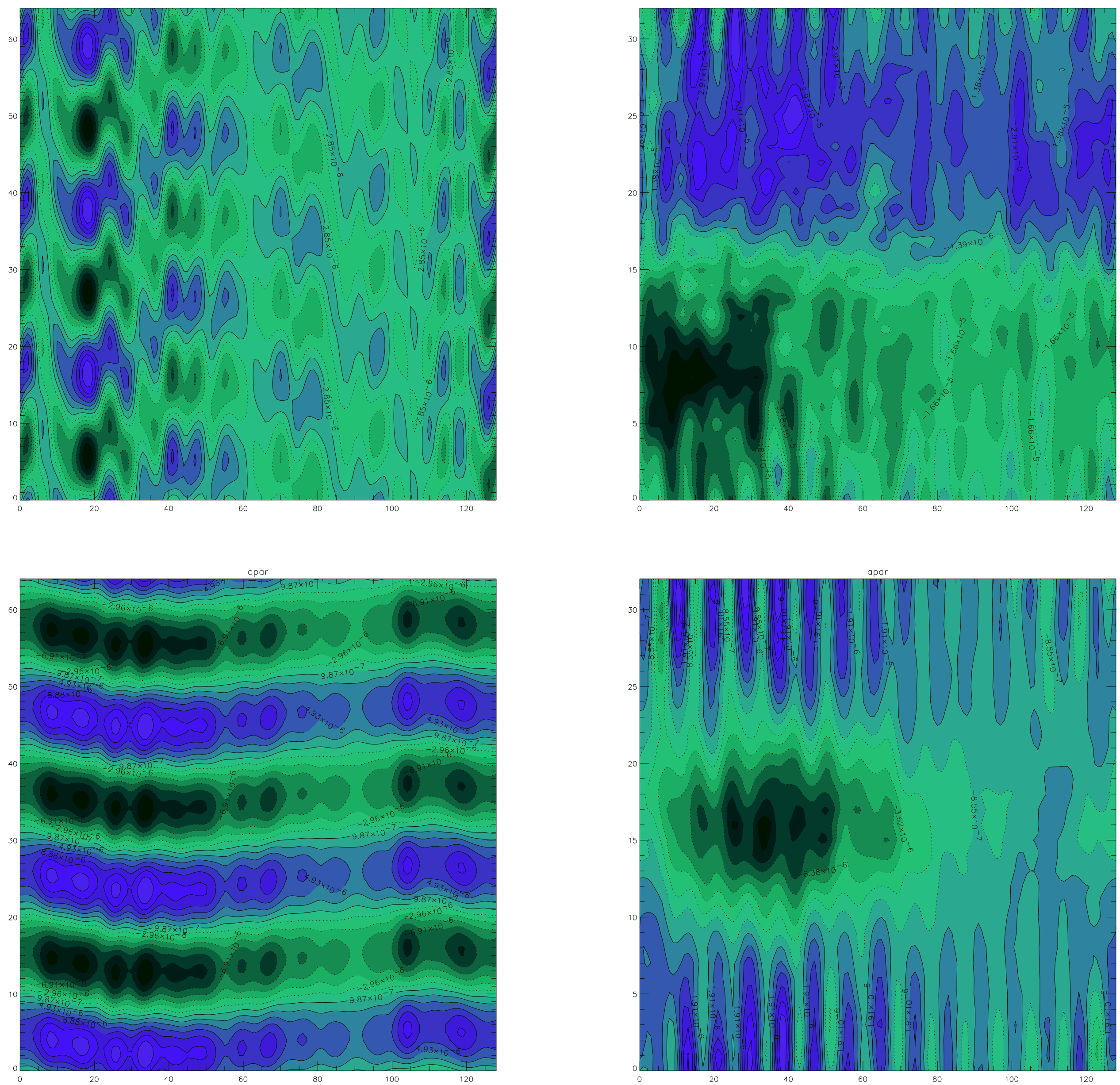
AITG $\beta_e = 1.25 \times 10^{-2}$ $\phi(x, y)$ and $\phi(x, z)$





$A_{\parallel}(x, y)$ and $A_{\parallel}(x, z)$

AITG $\beta_e = 1.5 \times 10^{-2}$ $\phi(x, y)$ and $\phi(x, z)$



$A_{\parallel}(x, y)$ and $A_{\parallel}(x, z)$

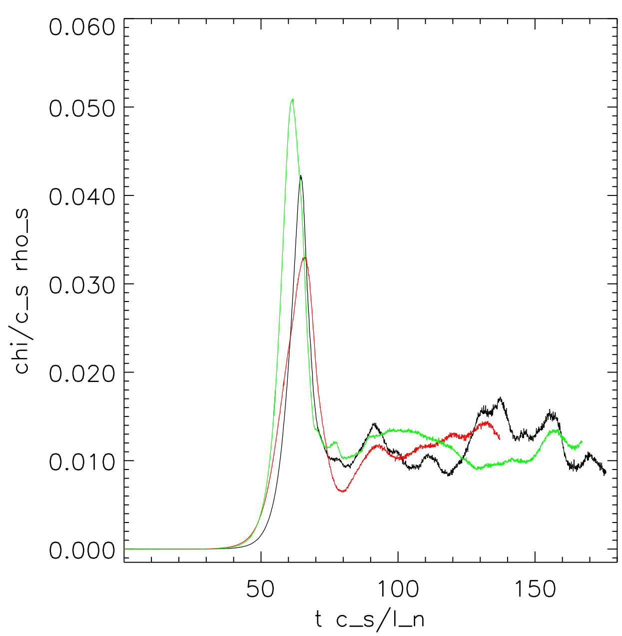
Results of Nonlinear Toroidal Simulations at Low β

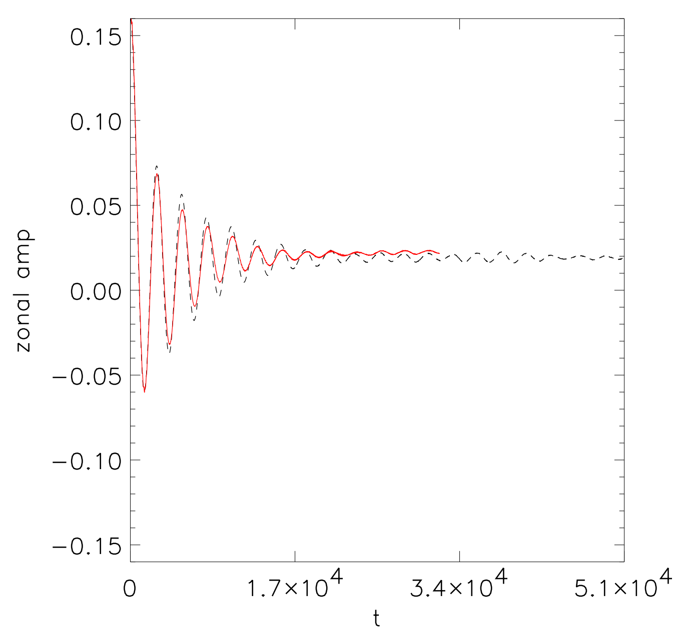
- DIII-D Base Case: $R_0\kappa_{\mathrm{Ti}} = 6.9$, $R_0\kappa_n = 2.2$, $T_i = T_e$, $r_0/R_0 = 0.18$, $q_0 = 1.4$, $\hat{s} = (r_0/q_0)(dq/dr) = 0.78$. Size of flux tube $64\rho_i \times 64\rho_i \times 2\pi q_0 R_0$, grid resolution 64×64, 4 194 304 ions and equal number of electrons. $\beta = 10^{-4}$. $\Omega_i \triangle t = 4$, $\varepsilon_g = 0.5$. ($\kappa_{\mathrm{Te}} = 0$).

- Growth rate is increased from adiabatic electron simulations by a factor of ~ 2, mainly due to trapped electrons. Ion heat flux increases by a factor of ≥ 5.

- e-i collisions reduces the growth rate and transport.

- Collisionless damping of zonal flow and the residual level are about the same





Summary and Future Work

- Linear electromagnetic simulation of microinstabilities with kinetic electrons using the split-weight scheme is extended from the regime of $\beta_e \frac{m_i}{m_e} \leq 1$ to $\beta_e \frac{m_i}{m_e} \geq 20$, for instabilities on the ion Larmor radius scale.
- The flux-tube based code includes both passing and trapped drift-kinetic electrons, gyrokinetic ions, electron-ion collisions, ion-ion collisions (the drag-diffusion part).
- Electron-ion collisions are strongly stabilizing for the Kinetic Ballooning Modes. Mode structure changes as β passes a critical value.

Future Work

- Signal of the low frequency instability of interest is contaminated by the high frequency Alfven noise.
  - Are Alfven frequency fluctuations present in turbulence?
  - If not, how to avoid this in simulations?
- Simulations at moderate to high β do not saturate. With linear electrons, saturation level too high.
  - What is the saturation mechanism for finite β?